

MEMORANDUM

Date: November 28, 2018
To: Dr. Hema Ramsurn
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Subject: Mixing Chamber Design Project

Introduction: Task & Givens

Task: design a process incorporating an insulated mixing chamber that uses any combination of four predefined source water streams to output a stream at temperature $T_e = 70^\circ\text{C}$ and mass flow rate $\dot{m}_e = 5 \text{ kg/s}$. Consider at least three solution scenarios; choose the one that minimizes entropy generation rate \dot{S}_{gen} . Temperature T and phase are given for all streams (organized in Table 1, Appendix 1). Pressure $P = 1 \text{ atm} = 101.325 \text{ kPa}$ for all streams.

Analysis

For simplicity an insulated mixing chamber is the only engineering device used. System boundary definition: fixed volume of mixing chamber represented by dotted green line in Figure 1. Heat transfer is negligible because mixing chamber is insulated, i.e. adiabatic system with $\dot{Q} = 0$.

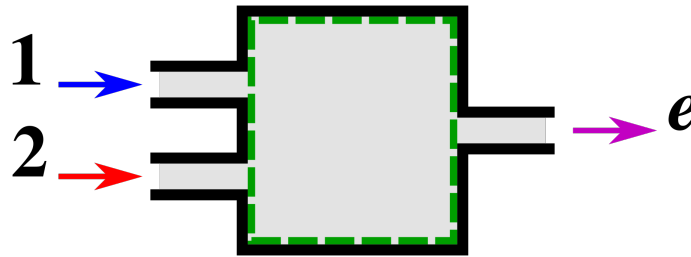


FIGURE 1
Mixing chamber schematic diagram

For simplicity, only two source streams are used per solution scenario. In this context of a pairing of two source streams for a solution scenario, the one at lesser T will be denoted by the number 1 in subscripts and Figure 1; the one at greater T will be denoted by the number 2. Pairings are restricted by: $T_1 < T_e < T_2$. Four viable pairings considered: 1-3, 1-4, 2-3, 2-4. For simplicity the sum of the areas of the inlets is equal to the area of the outlet; $A_1 + A_2 = A_e$.

For simplicity the process is made steady state. Mass flow balance: $\Sigma \dot{m}_i = \Sigma \dot{m}_e$; for two inlets and one exit:

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_e \quad (1)$$

Inlet-outlet change in kinetic energy $\Delta\text{KE} = 0$, and change in potential energy $\Delta\text{PE} = 0$ because changes in fluid velocity and elevation are negligible. Energy loss due to flow friction is assumed to be negligible. With these restrictions, the process is isenthalpic. Enthalpy flow balance: $\Sigma \dot{m}_i h_i = \Sigma \dot{m}_e h_e$; for two inlets and one exit:

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_e h_e \quad (2)$$

2nd law of thermodynamics for steady state adiabatic process: $\Sigma \dot{m}_e s_e - \Sigma \dot{m}_i s_i = \dot{S}_{\text{gen}}$; for two inlets and one exit:

$$\dot{m}_e s_e - \dot{m}_1 s_1 - \dot{m}_2 s_2 = \dot{S}_{\text{gen}} \quad (3)$$

Solution

Equations (1), (2), and (3) are regarded as a system of equations. Given T and P for all streams supply the two independent intensive values required to define states. Specific enthalpy h and specific entropy s values for all streams are resolved using the data tables in our textbook, then summarized in Table 2 (Appendix 1). Table 3 below summarizes the remaining three unknowns (\dot{m}_1 , \dot{m}_2 , and \dot{S}_{gen}) resolved for the four source pairings, making it clear that the pairing of sources 1 and 4 minimizes \dot{S}_{gen} at 9.10343 kJ/K. See Appendix 1 for detailed calculations.

TABLE 3
System of equations resolved for the four source pairings

Sources Used				\dot{m}_1 (kg/s)	\dot{m}_2 (kg/s)	\dot{S}_{gen} (kJ/K)
1	2	3	4			
X		X		1.671	3.329	0.1178
X			X	4.603	0.397	0.4476
	X	X		2.782	2.218	0.0441
	X		X	4.833	0.167	0.1284

Conclusion

From our calculations utilizing justified assumptions (see appendix 1), we have concluded the mixing scenario with the lowest rate of entropy generation is for inlet 1 to carry source 2 with a mass flow rate of 2.78 kg/s and for inlet 2 to carry source 3 with a mass flow rate of 2.21 kg/s. This result is as expected given our acquired intuition about entropy generation. A process with the fewest irreversibilities, such as the mixing of two fluids that each began at internal equilibrium, consequently has the lowest rate of entropy generation. Should this project take place in the real world, we would not recommend using source 4, as it would require a greater heat input to maintain and superheated steam can be corrosive. Our lack of recommendation for using source 1, despite its availability due to being close to room temperature, is because neither combinations that used source 1 had the lowest entropy generation. We would recommend our given solution, which has lowest rate of entropy generation, because of its feasibility and cost effectiveness.

Appendix 1: Calculations

Givens

TABLE 1
Givens for all streams arranged by ascending T

Stream	Phase	T ($^{\circ}\text{C}$)
Source 1	liquid water	20
Source 2	liquid water	50
Exit	liquid water	70
Source 3	liquid water	95
Source 4	superheated steam	120

Finding h and s

For compressed liquid states at P only slightly greater than P_{sat} , properties are close enough for our purposes to those at P_{sat} . Therefore saturated water table A-4 in our textbook is used to find h and s for sources 1 through 3 and exit; no interpolation necessary.

Source 4 in superheated steam phase required the use of table A-6 and interpolation. Suppose we wanted to find $h(120^{\circ}\text{C}, 101.325 \text{ kPa})$. We would use linear interpolation, beginning with the point-slope formula for a line:

$$\begin{aligned}y - y_1 &= m(x - x_1) \\ &\Downarrow \\ y &= \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1) + y_1 \quad (4) \\ &\Downarrow\end{aligned}$$

$$\begin{aligned}h(120^{\circ}\text{C}, 101.325 \text{ kPa}) &= \frac{(2776.6 \text{ kJ/kg} - 2675.8 \text{ kJ/kg})}{(150^{\circ}\text{C} - 100^{\circ}\text{C})} (120^{\circ}\text{C} - 100^{\circ}\text{C}) + 2675.8 \text{ kJ/kg} \\ &= 2716.1 \text{ kJ/kg}\end{aligned}$$

Definition of new function for assistance with interpolation modeled after (4):

$$\text{map}(a, b, c, p, q) = \frac{(q - p)}{c - a}(b - a) + p,$$

The map function can be used for both interpolation and extrapolation. It maps the range from a to c onto the range from p to q and returns a value that is positioned relative to p and q in proportion to how the value b is positioned relative to a and c . All code that follows is intended for use with “GNU Octave”, a free and open-source MatLab equivalent.

Equivalent function definition in Octave:

```
function retval = map (a,b,c,p,q)
retval = (((b-a)*(q-p))/(c-a))+p; # retval is returned value
endfunction
```

Sample usage:

```
>> map(1,2,3,0,100)
ans = 50
>> map(1,0,3,0,100) # extrapolation demonstration
ans = -50
```

Hypothetical assumption: $h(120^\circ\text{C}, 101.325 \text{ kPa}) = h(120^\circ\text{C}, 0.10 \text{ MPa})$ close enough for our purposes. Solving for $h(120^\circ\text{C}, 0.10 \text{ MPa})$ again, this time using the map function:

$$\begin{aligned} h(120^\circ\text{C}, 0.10 \text{ MPa}) &= \text{map}(100^\circ\text{C}, 120^\circ\text{C}, 150^\circ\text{C}, \\ &\quad 2675.8 \text{ kJ/kg}, 2776.6 \text{ kJ/kg}) \\ &= 2716.1 \text{ kJ/kg} \end{aligned}$$

Octave equivalent:

```
>> map( 100, 120, 150, 2675.8, 2776.6 )
ans = 2716.1
```

Now: interpolation between different P values as well as T . It is helpful to think of it as double interpolation or nested interpolation, where one of the map function's instances is actually for extrapolation.

$$\begin{aligned} h(120^\circ\text{C}, 101.325 \text{ kPa}) &= \text{map}(0.10 \text{ MPa}, 0.101325 \text{ MPa}, 0.20 \text{ MPa}, \\ &\quad h(120^\circ\text{C}, 0.10 \text{ MPa}), h(120^\circ\text{C}, 0.20 \text{ MPa})) \\ &= \text{map}(0.10 \text{ MPa}, 0.101325 \text{ MPa}, 0.20 \text{ MPa}, \\ &\quad \text{map}(100^\circ\text{C}, 120^\circ\text{C}, 150^\circ\text{C}, \\ &\quad 2675.8 \text{ kJ/kg}, 2776.6 \text{ kJ/kg}), \\ &\quad \text{map}(120.21^\circ\text{C}, 120^\circ\text{C}, 150^\circ\text{C}, \\ &\quad 2706.3 \text{ kJ/kg}, 2769.1 \text{ kJ/kg})) \\ &= 2716.3 \text{ kJ/kg} \end{aligned}$$

This number is not very different from that derived by assuming $P = 0.10 \text{ MPa}$, bringing into question the utility of double interpolation as opposed to single interpolation for this application.

Octave equivalent:

```
>> map( .1, .101325, .2, map( 100, 120, 150, 2675.8, 2776.6 ), map( 100, 120,
150, 2706.3, 2769.1 ) )
ans = 2716.3
```

Specific entropy single interpolation:

$$\begin{aligned} s(120^\circ\text{C}, 0.20 \text{ MPa}) &= \text{map}(100^\circ\text{C}, 120^\circ\text{C}, 150^\circ\text{C}, \\ &\quad 7.3611 \text{ kJ/kg}\cdot\text{K}, 7.6148 \text{ kJ/kg}\cdot\text{K}) \\ &= 7.4626 \text{ kJ/kg}\cdot\text{K} \end{aligned}$$

Octave equivalent:

```
>> map( 100, 120, 150, 7.3611, 7.6148 )
ans = 7.4626
```

Specific entropy double interpolation:

$$\begin{aligned}
 s(120^\circ\text{C}, 101.325 \text{ kPa}) &= \text{map}(0.10 \text{ MPa}, 0.101325 \text{ MPa}, 0.20 \text{ MPa}, \\
 &\quad s(120^\circ\text{C}, 0.10 \text{ MPa}), s(120^\circ\text{C}, 0.20 \text{ MPa})) \\
 &= \text{map}(0.10 \text{ MPa}, 0.101325 \text{ MPa}, 0.20 \text{ MPa}, \\
 &\quad \text{map}(100^\circ\text{C}, 120^\circ\text{C}, 150^\circ\text{C}, \\
 &\quad 7.3611 \text{ kJ/kg}\cdot\text{K}, 7.6148 \text{ kJ/kg}\cdot\text{K}), \\
 &\quad \text{map}(120.21^\circ\text{C}, 120^\circ\text{C}, 150^\circ\text{C}, \\
 &\quad 7.1270 \text{ kJ/kg}\cdot\text{K}, 7.2810 \text{ kJ/kg}\cdot\text{K})) \\
 &= 7.4589 \text{ kJ/kg}\cdot\text{K}
 \end{aligned}$$

Octave equivalent:

```
>> map( .1, .101325, .2, map( 100, 120, 150, 7.3611, 7.6148 ), map( 100, 120,
150, 7.1270, 7.2810 ) )
ans = 7.4589
```

While it is simpler to use single interpolation and the benefit to double interpolation is marginal we can afford to perform double interpolation, so we did, so we use the results it gives moving forward.

TABLE 2
State variables for all streams, sorted by ascending T

Stream	T ($^\circ\text{C}$)	h (kJ/kg)	s (kJ/kg·K)
Source 1	20	83.915	0.2965
Source 2	50	209.34	0.7038
Exit	70	293.07	0.9551
Source 3	95	398.09	1.2504
Source 4	120	2716.3	7.4589

Solving System of Equations

The following system of equations consists of three equations (mass balance, enthalpy balance, and 2nd law) and a total of three unknowns (two inlet mass flow rates and entropy generation rate, i.e. \dot{m}_1 , \dot{m}_2 , and \dot{S}_{gen} .) It is manipulated into a form suitable for Gauss-Jordan elimination then solved by that method.

$$\begin{cases}
 \dot{m}_1 + \dot{m}_2 = \dot{m}_e \\
 \dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_e h_e \\
 \dot{m}_e s_e - \dot{m}_1 s_1 - \dot{m}_2 s_2 = \dot{S}_{\text{gen}}
 \end{cases}$$

↓

$$\begin{cases} (1)\dot{m}_1 + (1)\dot{m}_2 + (0)\dot{S}_{\text{gen}} = (\dot{m}_e) \\ (h_1)\dot{m}_1 + (h_2)\dot{m}_2 + (0)\dot{S}_{\text{gen}} = (\dot{m}_e h_e) \\ (-s_1)\dot{m}_1 + (-s_2)\dot{m}_2 + (-1)\dot{S}_{\text{gen}} = (-\dot{m}_e s_e) \end{cases}$$

$$\begin{aligned} & \Downarrow \\ & \text{rref} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 5 \\ h_1 & h_2 & 0 & 5 \times 293.07 \\ s_1 & s_2 & 1 & 5 \times 0.9551 \end{array} \right) \\ & = \\ & \left(\begin{array}{ccc|c} 1 & 0 & 0 & \dot{m}_1 \\ 0 & 1 & 0 & \dot{m}_2 \\ 0 & 0 & 1 & \dot{S}_{\text{gen}} \end{array} \right) \end{aligned}$$

where \dot{m}_1 , \dot{m}_2 , and \dot{S}_{gen} are resolved

Octave rref() commands:

Sources 1 and 3:

```
>> rref( [ 1, 1, 0, 5; 83.915, 398.09, 0, 5*293.07; 0.2965, 1.2504, 1, 5*0.9551 ] )
ans =
1.00000 0.00000 0.00000 1.67136
0.00000 1.00000 0.00000 3.32864
0.00000 0.00000 1.00000 0.11781
```

Sources 1 and 4:

```
>> rref( [ 1, 1, 0, 5; 83.915, 2716.3, 0, 5*293.07; 0.2965, 7.4589, 1, 5*0.9551 ] )
ans =
1.00000 0.00000 0.00000 4.60273
0.00000 1.00000 0.00000 0.39727
0.00000 0.00000 1.00000 0.44757
```

Sources 2 and 3:

```
>> rref( [ 1, 1, 0, 5; 209.34, 398.09, 0, 5*293.07; 0.7038, 1.2504, 1, 5*0.9551 ] )
ans =
1.00000 0.00000 0.00000 2.78199
0.00000 1.00000 0.00000 2.21801
0.00000 0.00000 1.00000 0.04413
```

Sources 2 and 4:

```
>> rref( [ 1, 1, 0, 5; 209.34, 2716.3, 0, 5*293.07; 0.7038, 7.4589, 1, 5*0.9551 ] )
ans =
1.00000 0.00000 0.00000 4.83300
0.00000 1.00000 0.00000 0.16700
0.00000 0.00000 1.00000 0.12843
```

Appendix 2: Reference Data Tables

See following pages for exact tables used from the textbook.